effects are at the limit of observational accuracy and can be neglected in the present context.

Finally, it must be emphasized that this work, like that of Ref. 1, applies only to adiabatic-wall boundary layers; in the presence of a heat-transfer rate Q_w, Eq. (2) contains the additional parameter $Q_w/(\rho u_\tau^3)$, and the intermediate-temperature correlation is even more suspect. We cannot confidently extend Eq. (4), or any other formula, without measurements of skin friction in the presence of heat transfer.

References

[1] Allen, J. M., "Evaluation of Preston Tube Calibration Equations in Supersonic Flow," *AIAA Journal*, Vol. 11, No. 11, Nov. 1973, pp. 1461–1462.

[2] Allen, J. M., "Evaluation of Compressible-Flow Preston Tube Calibrations," TN D-7190, 1973, NASA.

[3] Patel, V. C., "Calibration of the Preston Tube and Limitations on its Use in Pressure Gradients," *Journal of Fluid Mechanics*, Vol. 23, 1965, pp. 185–208.

[4] Bradshaw, P. and Unsworth, K., "A Note on Preston Tube Calibrations in Compressible Flow," Aero Rept. 73-07, 1973, Imperial College, London, England.

[5] Coles, D. E. and Hirst, E. A., "A Young Person's Guide to the Data," *Proceedings, Computation of Turbulent Boundary Layers, 1968 AFOSR-IFP-Stanford Conference*, Vol. 2, Thermosciences Div., Stanford University, Stanford, Calif., 1969, pp. 1–45.

Reply by Author to P. Bradshaw and K. Unsworth

Jerry M. Allen*

NASA Langley Research Center, Hampton, Va.

I WISH to thank Bradshaw and Unsworth for their interest in my paper[1] and for using my data in developing their calibration equation. In order to assess their equation on the same basis as the ones evaluated in my paper, I have compared the skin friction determined from their equation [Eq. (4) of their paper] with measured skin friction in a manner similar to the evaluations performed in my Synoptic, and would like to present my findings.

First of all, a few words about the intermediate temperature concept. I recognize that this concept is not theoretically exact; however, it has proven very useful in obtaining engineering estimates of compressibility effects in zero pressure gradient boundary-layer prediction techniques. Because of its inexact theoretical basis, the usefulness of the intermediate temperature concept depends on its demonstrated validity over a wide range of test conditions, which was, indeed, the reason for generating the large volume of data in my study. Because my calibration equation (see Fig. 4 of my Synoptic) is based on the intermediate temperature concept, I recognize that it is probably invalid in strong pressure gradient flows; however, I feel it has a demonstrated validity in zero pressure gradient flow as shown in Fig. 5 of my Synoptic.

Figure 1 was prepared to show how the skin friction determined from the Bradshaw-Unsworth equation compares with measured skin friction for the three independent sets of supersonic calibration data used to evaluate the various equations in my Synoptic. The Bradshaw-Unsworth equation provides a very good fit to my data—which is to be expected since it was derived from my data—whereas my equation, being derived from all three sets of data, provides a slightly better fit to the total data available, as seen in Fig. 5 of my Synoptic. The differences in accuracy between the two equations, however, are relatively minor.

One of the principal conclusions of my paper was that valid results were obtained with large diameter tubes ($D/\delta > 0.2$) even though, as Bradshaw and Unsworth state, "one cannot expect law-of-the-wall arguments to be valid for such large tubes. . . ." Figure 1 shows that the Bradshaw-Unsworth equation also gives large-tube results which compare favorably with the smaller-tube results. The reason that these large tubes give valid results can be seen in Fig. 2 in which measured velocity profiles obtained with large and small tubes are compared to the true profile (obtained in Ref. 4 by extrapolation to $D = 0$). Although the traverse of the large tube through the boundary layer produced a severe distortion of the measured profile compared to the true

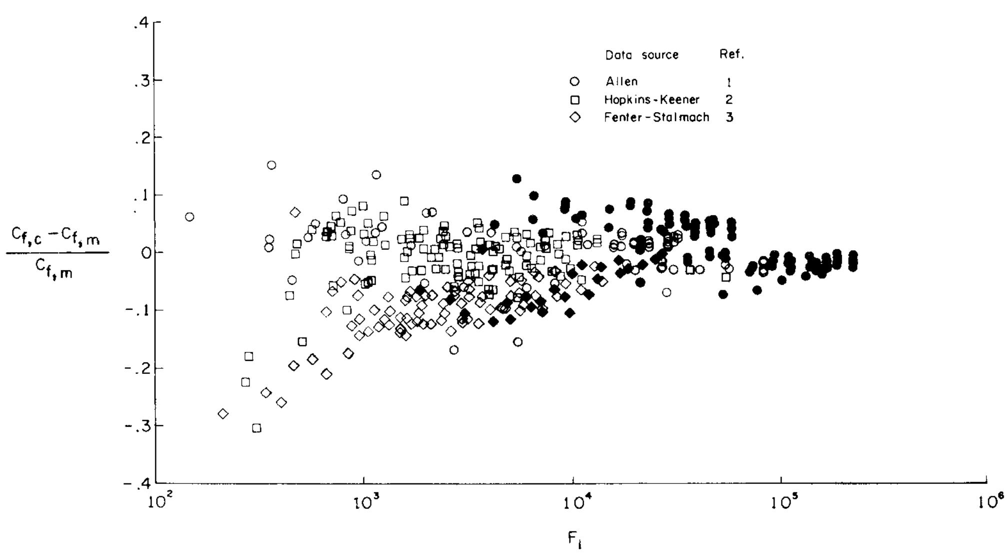

Fig. 1 Comparison between measured friction and friction determined from Bradshaw-Unsworth equation. Solid symbols denote data for $D/\delta > 0.2$.

Received March 11, 1974.

Index category: Boundary Layers and Convective Heat Transfer—Turbulent.

* Research Scientist, High Speed Aircraft Division. Member AIAA.

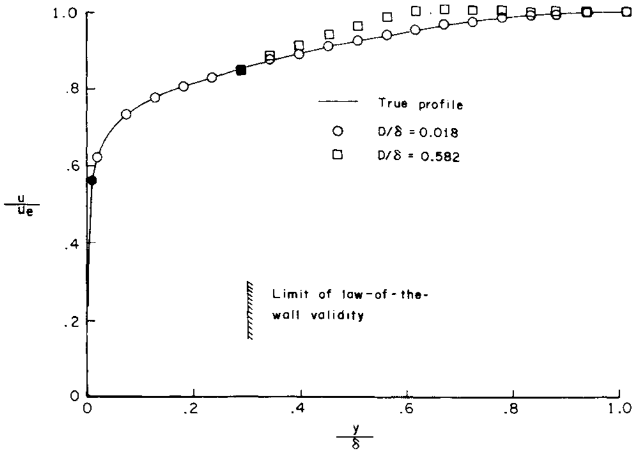

Fig. 2 Effect of tube size on velocity measurement for $M_e = 1.975$, $R_\theta = 3.6 \times 10^4$, and $T_t = 41°C$. Solid symbols denote data obtained with tube touching test surface.

profile, the large tube resting on the test surface (that is, acting as a Preston tube) measured a velocity very close to the true value. Since the law-of-the-wall region of this true profile extends to about $y/\delta = 0.3$, the data from this large Preston tube gives results which are valid in a law-of-the-wall context. In fact, the three sets of Preston tube data shown in Fig. 1 give more consistent results at the larger values of F_1; that is, at the larger Preston tube diameters.

There is one minor correction to the Bradshaw-Unsworth paper I would like to make. They state that my data for $D/\delta > 0.2$ were omitted from their Fig. 1. Since data for $D/\delta > 0.2$ were in fact included in this figure, I presume that they meant that data for $D/\delta > 0.2$ were omitted from the crossplots contained in their Fig. 2. This correction has no effect on their resulting calibration equation.

In summary, I feel that either my equation or the Bradshaw-Unsworth equation can be used to obtain comparably valid Preston tube skin-friction results in small pressure gradient adiabatic turbulent boundary layers up to at least high supersonic speeds, and that neither equation is restricted to small tube sizes. There are, however, three important differences between the two calibration equations of which the user should be aware: 1) my equation requires a knowledge of boundary-layer-edge Mach number whereas the Bradshaw-Unsworth equation is based entirely on wall conditions; 2) my equation is directly solvable for skin friction whereas the Bradshaw-Unsworth equation requires iteration; and 3) my equation is probably restricted to small pressure gradient flows whereas the Bradshaw-Unsworth equation is potentially valid for small probe sizes in pressure gradient flows, although no reliable data at present exists to document this validity.

References

[1] Allen, J. M., "Evaluation of Compressible-Flow Preston Tube Calibrations," TN D-7190, 1973, NASA.

[2] Hopkins, E. J. and Keener, E. R., "Study of Surface Pitots for Measuring Turbulent Skin Friction at Supersonic Mach Numbers—Adiabatic Wall," TN D-3478, 1966, NASA.

[3] Fenter, F. W. and Stalmach, C. J., Jr., "The Measurement of Local Turbulent Skin Friction at Supersonic Speeds by Means of Surface Impact Pressure Probes," DRL-393, CM-878 (Contract NOrd-16498), Oct. 21, 1957, University of Texas, Austin, Texas.

[4] Allen, J. M., "Impact Probe Displacement in a Supersonic Turbulent Boundary Layer," *AIAA Journal*, Vol. 10, No. 4, April 1972, pp. 555–557.

Vinti Solution for Free-Flight Rocket Trajectories

Donald van Z. Wadsworth*
Bell Telephone Laboratories, Holmdel, N.J.

THERE have been a number of inquiries concerning difficulties in programing the formulations in my article.[1] The problems are caused by typographical errors, for which corrigenda are given below, and by mathematical singularities or indeterminate forms which cause computational loss of accuracy in the case of certain pathological trajectories (i.e., near-polar and near-equatorial orbits) and in the vicinity of apogee, perigee, or maximum (minimum) declination. Difficulties of the latter type also occur in the Keplerian formulations and must be handled by analytically removing the apparent singularity or indeterminate form (including differences of large, nearly equal, quantities) prior to numerical computation.

As an example of indeterminacy, in the aforementioned article, the equation for the right ascension contains a factor $c_3(1-\eta_a{}^2)^{-1/2}$ which behaves as the indeterminate form 0/0 in the case of polar orbits since c_3 is the Vinti equivalent of the polar component of angular momentum and η_a is the Vinti equivalent of the sine of the orbital inclination angle. By expanding η_a in terms of the small parameter k_0, retaining only terms to the order of $k_0{}^2$ consistent with other expansions in the article, the indeterminacy is removed

$$(c_3/c_2)(1-\eta_a{}^2)^{-1/2} \simeq 1-(k_0/2)(1-c_3{}^2/c_2{}^2) \times [1+(k_0/4)(1-5c_3{}^2/c_2{}^2)]$$

where c_2 is the angular momentum.

The following are the corrigenda where *l.* denotes line; headings and each line of an equation are counted as one line.

Page 1351, Col. 2: *l.* 20 in the expression for U the multiplier of the $P_4(\ )$ term should be c^4R^{-4}.

Page 1352, col. 2: *l.* 26, insert a minus sign inside the square root; *l.* 41, change the middle plus sign to minus in the $P(\ )$ equation; *l.* 43, insert a left-hand bracket immediately before the Π in the right-hand side of the $R(\ )$ equation and a corresponding right-hand bracket at the end of the equation; *l.* 48, change κ to lower-case Roman k (with subscript 0).

Page 1353, col. 2: *l.* 28, in the second line of the s^2P equation, change the first minus to a plus sign and replace the 3 with a 6. The last factor, $F(\ )$, should be replaced by the factor

$$\{F(f,\kappa_1)-[F(f,\kappa_1)-E(f,\kappa_1)]\kappa_1{}^{-2}\}$$

Two higher order terms should be added to the right side of the s^2P equation (an omission pointed out by M. V. Boelitz of the Raytheon Co.)

$$+(2\varepsilon^2/3)(\Delta/s-2c^2/s^2)\sin f(3-\sin^2 f)$$
$$-(\varepsilon^4/8)(c^2/s^2)(3f+5\sin f\cos f-2\sin^3 f\cos f)$$

Page 1353, col. 2: *l.* 48, in the equation for c_1, the 1 subscript should obviously be an $_i$.

Page 1354, col. 1: *l.* 18 and *l.* 19, all the arguments of the sgn functions should have subscripts i; *l.* 29, the missing left side of the equation is f.

Page 1354, col. 2: *l.* 40, the correct relation for the azimuth angle is

$$\cos\beta = (R\dot{Z}-Z\dot{R})(V\cos\gamma)^{-1}(X^2+Y^2)^{-1/2}$$

References

[1] Wadsworth, D. Z., "Vinti Solution for Free-Flight Rocket Trajectories," *AIAA Journal*, Vol. 1, No. 6, June 1963, pp. 1351–1354.

Received February 25, 1974.
Index category: Earth-Orbital Trajectories.
* Department Head.